

Emotional equations

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1. Introduction

Emotions are sometimes portrayed as phenomena that resist theoretical treatment, especially of a mathematical kind. In this paper, I argue that this portrayal is incorrect. In particular, so-called “social preference models” such as the inequity aversion model of Ernst Fehr and Klaus Schmidt , offer one persuasive view of how we might construct a mathematical theory of emotion. It would therefore appear that psychological science has now advanced to the point at which we can begin to quantify emotions in a precise scientific way by means of equations This will helps to demystify the emotions, and help us see them as measurable psychological variables capable of deep theoretical treatment like any other.

I begin by discussing some empirical findings from game theory. I then go on to show how these findings can be accommodated in game theoretic equations by introducing parameters that designate emotions such as envy and guilt. Finally, I suggest that these models may also have a practical value in addition to their scientific merit, in that they help to challenge the pernicious norm of self-interest.

2. The ultimatum game: theory and evidence

One of the greatest scientific achievements of the twentieth century was the development of game theory . Game theory analyses behaviour in strategic situations, in which an individual's success in making choices depends on the choices of others. The power of game theory lies in its generality and mathematical precision. It provides a parsimonious and elegant framework in which every conceivable game can be captured and analysed.

One of the most well-studied games in game theory is the ultimatum game . In this game, two strangers are paired up and given a sum of money. One of them – usually referred to as the “proposer” – has to decide how to divide up the money. The proposer might suggest a 50-50 split, or she might be mean and offer only 10 per cent to the other person. The other player – usually referred to as the “responder” – can either accept this offer or reject it. If the responder accepts the offer, each player walks away with their share of the money. If he rejects the offer, each player walks away with nothing.

Game theory predicts that the proposer should always offer the smallest amount possible. It also predicts that the responder should always accept the proposer's offer, no matter how small it is. After all, some money is always better than none.

But this isn't what people actually *do* when they play this game. Instead of offering the smallest possible amount, most proposers offer between 40 and 50

per cent of the money. And on the few occasions that proposers offer less than 20 per cent, responders reject about half of those offers.

These findings have been replicated many times in studies all around the world. It's an almost universal pattern. The only odd ones out are people from preliterate cultures. They are the only people who seem to behave as game theory predicts .

3. Social preferences

Many people have interpreted the experimental findings discussed in the previous section as showing that people are irrational . The argument seems simple. Game theory tells us how rational people will play the ultimatum game. But people don't play the ultimatum game this way. Therefore, it follows that people do not behave rationally when playing the ultimatum game.

However, the first premise in that argument is not true. Game theory does not in fact tell us how rational people play games. It tells us how *rational self-interested* people play games. So the fact that people don't play the ultimatum game in the way predicted by game theory tells us that either (1) people are not entirely rational or (2) people are not entirely self-interested.¹

Theorists differ in which of these conclusions they prefer. Those who prefer to retain the self-interest assumption must accordingly relax the rationality assumption. Theorists who follow this route have developed what they call "learning models" . In this paper, however, my focus is on a different family of models called "social preference models". These models retain the rationality assumption and relax the self-interest assumption.

Social preference models, in other words, retain the basic assumption of rational choice theory – namely, that people make decisions in a way that tends to maximise their utility. What these models add to this is the idea that utility is now a function not only of one's own personal gain but also of how much other people gain. In other words, these models propose that people care about other things in addition to their own personal gains and losses – things like fairness and equality.

We can express this idea mathematically like this (equation 1):

Equation 1:
$$U_i = U_i(\pi_i, \pi_{-i})$$

Social preference models differ from one another in the ways that they specify exactly *how* one's utility depends on the gains and losses of others.

One influential social preference model is the inequity aversion model of Ernst Fehr and Klaus Schmidt . Inequity aversion means that people resist inequitable outcomes. In other words, people are willing to give up some material payoff to

¹ A third conclusion is logically possible – namely, that people are neither rational nor self-interested. To accept this conclusion would entail rejecting game theory as having any applicability to human behaviour.

shift their group or society in the direction of more equitable outcomes. The theory is expressed in terms of equation 2, in which there are two parameters, one for envy and one for guilt: α = envy, and β = guilt.

Equation 2: The inequity aversion model of Fehr and Schmidt

$$U_i(x) = x_i - \frac{1}{n-1} \left[\sum_{j \neq i} \alpha_i \max[x_j - x_i, 0] + \sum_{j \neq i} \beta_i \max[x_i - x_j, 0] \right], i \neq j.$$

$x = x_1, \dots, x_n$ vector of monetary payoffs. $\alpha_i > \beta_i \geq 0$.

In plain English, this equation states that my utility is decreased when others have more than me in proportion to the extent that I feel envy. My utility is also decreased when others have less than me in proportion to the extent that I feel guilt.

Fehr and Schmidt found that their equation could explain a lot of the data from ultimatum game experiments if they assumed that values of these two parameters were distributed according to the values in table 1:

Table 1: From Fehr and Schmidt 1999:

ASSUMPTIONS ABOUT THE DISTRIBUTION OF PREFERENCES					
DISTRIBUTION OF α 'S AND ASSOCIATED ACCEPTANCE THRESHOLDS OF BUYERS			DISTRIBUTION OF β 'S AND ASSOCIATED OPTIMAL OFFERS OF SELLERS		
$\alpha = 0$	30 percent	$s'(0) = 0$	$\beta = 0$	30 percent	$s^* = 1/3$
$\alpha = 0.5$	30 percent	$s'(0.5) = 1/4$	$\beta = 0.25$	30 percent	$s^* = 4/9$
$\alpha = 1$	30 percent	$s'(1) = 1/3$	$\beta = 0.6$	40 percent	$s^* = 1/2$
$\alpha = 4$	10 percent	$s'(4) = 4/9$			

In other words, equation 2 predicts actual behaviour in the ultimatum game if about 30 per cent of people feel no envy and about 10 per cent are very envious.

The most important thing about Fehr and Schmidt's work is not, however, the specific values for the envy and guilt terms that they derive from their study of the experimental results, interesting as these are. Rather, it is the very idea that emotions can be described by terms in an equation that models a utility function.

Emotions are sometimes portrayed as phenomena that resist theoretical treatment, especially of a mathematical kind. I myself, for example, stated in the introduction to my book, *Emotion: The Science of Sentiment*, that I doubted we could ever "reduce emotional experience to a dry formula". Since discovering social preference models such as the inequity aversion model of Fehr and Schmidt, I have changed my mind. I now think that psychological science has advanced to the point at which we can begin to quantify emotions in a precise

scientific way by means of equations like those of Fehr and Schmidt. There is still a long way to go, of course, when it comes to constructing these equations and putting actual values on the parameters, but thanks to Fehr and Schmidt and others like them we can at least begin to glimpse what a mathematical theory of emotion should look like. And this in itself is no small achievement. It helps to demystify the emotions, and helps us see them as measurable psychological variables capable of deep theoretical treatment like any other.

4. The norm of self-interest

A cynic might argue that the results of ultimatum game experiments will come as a surprise only to economists, who have been brainwashed by their professional education into thinking that we are all selfish. But it isn't just economists who hold this view. According to the philosopher Peter Singer, most people in the West "are in thrall to the idea that it is 'normal' to be self-interested". For example, even when people act altruistically, they offer self-interested explanations: they say "I volunteered because it got me out of the house" – not "because I wanted to help".

Singer's thesis is supported by some very interesting research done by Rebecca Ratner and Dale Miller on what they call "the norm of self-interest". This is the tendency to explain behaviour in terms of self-interest, even when it appears genuinely altruistic. Ratner and Miller found that when people's attitudes and actions do not conform to their self-interest, this can provoke severe psychological discomfort in actor and confusion and anger in observers.

The interesting thing about this norm is that it persists despite a lot of contrary evidence. There are many examples from everyday life that people are not entirely selfish creatures. People leave tips at restaurants to which they will never return. They donate blood to strangers although this will not increase their own chances of receiving blood when they need it. And they vote in elections even though the chance that their vote will make a difference is vanishingly small.

As Singer notes, the norm of self-interest is socially pernicious, because if we believe that no one else acts altruistically, we are less likely to do it ourselves. The norm thus becomes a self-fulfilling prophecy. This, then, is another reason for bringing the social preference models discussed in the previous section to a wider audience. Perhaps the scientific rigour of these models will make them more powerful antidotes to the ideology of self-interest than the examples from everyday life listed above.

5. Conclusion

Progress in science often consists in finding mathematical expressions for phenomena that were previously only described in verbal terms. Early psychologists such as Ernst Weber (1795–1878) and Gustav Fechner (1801–1887) recognised this, and modelled the relationship between the physical magnitudes of stimuli and their perceived intensity by means of differential equations. In the past few decades, however, psychologists have increasingly

retreated from these bold aspirations and contented themselves with more qualitative accounts. The mathematical tools employed by psychologists today are almost exclusively statistical in nature. Those who do still aspire to model psychological phenomena by means of equations come largely from other disciplines such as economics.

It is high time that psychologists regained their lofty aspirations to scientific rigour. To do this, they should acquaint themselves with the research on social preferences. This research provides a glimpse of what a mathematical theory of emotion should look like.

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References